

## Chapter 1 Basic Concepts

### 1.1 Interest

Interest is a fee that is charged for the use of someone else's money.

ex. Joe borrows \$2500 and repays the loan with 3 consecutive year end payments of \$900.

$$P = \$2500$$

$$A = \$900 \text{ per yr}$$

$$n = 3 \text{ yrs}$$

$$I = nA - P = 3(\$900) - \$2500 = \$200.$$

### 1.2 Interest Rate

The interest rate for a given period of time (day, month, yr) is the ratio of interest charged (or earned) to the starting balance or principal at the beginning of the period.

ex. Sue deposits \$1800. in a bank and withdraws \$1822.50 3 months later. Find the interest rate per quarter the bank pays. (Assume quarterly compounding)

$$i = \frac{I}{P} = \frac{\$1822.50 - \$1800.}{\$1800.} = 0.0125 \quad (1.25\% \text{ per qtr})$$

### 1.3 Simple Interest

An investment earns (loan is charged) simple interest when the formula for total interest is computed as

$$I = niP$$

$P$  = Principal

$i$  = interest rate

$n$  = number of interest periods

$I$  = Total Interest

ex. An account earns 5% per year. How much <sup>interest</sup> does a \$12,000 principal earn after 4 yrs of simple interest?

$$I = niP = (4 \text{ yrs}) (0.05 / \text{yr}) (\$12,000) = \$2400.$$

Note, if the number of interest periods double, the amount of simple interest earned will also double.

After "n" interest periods, the total interest plus principal is

$$F = P + I$$

$$= P + niP$$

$$= P(1 + ni)$$

### 4 Compound Interest

When interest is compounded, the future amount (interest plus principal) is calculated from

$$F = P(1 + i)^n$$



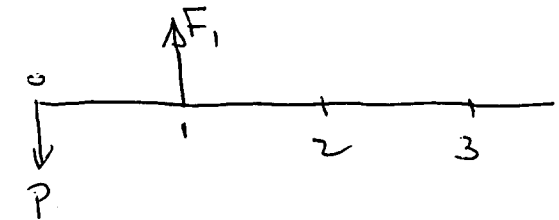
After 1 period,

$$I_1 = iP$$

$$F_1 = P + I_1$$

$$= P + iP$$

$$= P(1+i)$$



After 2 periods,

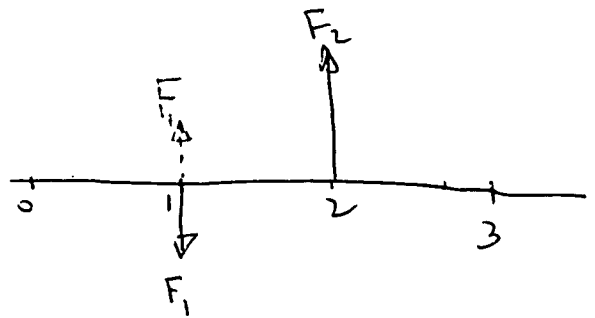
$$I_2 = iF_1$$

$$F_2 = F_1 + I_2$$

$$= \del{F_1} + iF_1$$

$$= F_1(1+i)$$

$$F_2 = P(1+i)^2$$



Note:

~~$F_2 = P(1+i)^2$~~   
 ~~$= P(1+i) + iP(1+i)$~~

$$F_2 = P(1+2i+i^2)$$

$$= P(1+2i) + i \cdot iP$$

$$= P(1+2i) + \underbrace{i I_1}_{\text{compound interest term}}$$

### Time Value of Money

Since money can earn interest, the value of \$1 now is more some time in the future

ex. For  $i = 0.08$  per yr

$$F = P(1+i)^n$$

$$= \$1(1.08)^n$$

n	Future Worth of \$1.00 <del>at time n</del> n yrs later
0	\$1.00
1	\$1.08
2	\$1.1664

Similarly \$1.00 in the future is equivalent to a lesser value in the present provided an interest earning opportunity exists.

ex For  $i = 0.10$

$$P = \frac{F}{(1+i)^n}$$

$$= \frac{\$1}{1.1^n}$$

Present worth of \$1.00	
$n$	$n$ yrs earlier
0	\$1.00
1	\$0.9090
2	\$0.8264
10	\$0.3855

## 6 Inflation

Inflation refers to the increase in the cost of goods and services from one year to the next.

ex  $\lambda =$  inflation rate = 0.04 per yr

PC = Present cost of auto is \$20,000

FC = Future cost of auto " $n$ " yrs later

$$FC = PC(1+\lambda)^n$$

$$= \$20,000(1.04)^n$$

$n$	FC
0	\$20,000
1	\$20,800
2	\$21,632.
10	\$29,605.



# Chap 4 Discrete & Periodic Compounding

## Nominal and Effective Interest Rates

When interest is compounded more frequently than once a year, there is a nominal interest rate and an effective interest rate.

ex. A credit card company charges interest on unpaid balances monthly. Suppose the rate is  $i = 1.5\%$  per month.

effective interest rate  $i = 1.5\%$  per month

nominal interest rate  $r = 12 \times 1.5\%$

$= 18\%$  per yr (compounded monthly)

when there are  $m$  interest periods per yr

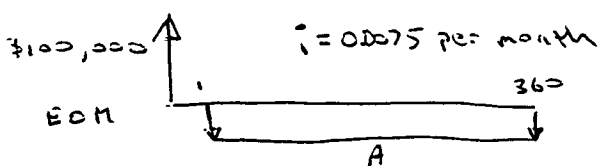
$$i = \frac{r}{m} \quad (\text{nominal int rate is usually per annum})$$

ex. Joe borrows \$100,000 from a bank advertising <sup>30 year</sup> mortgage loans at  $9\%$  per yr. ~~for 30 years.~~

Since the payments are made monthly, the effective monthly interest rate is required.

$$i = \frac{r}{m} = \frac{0.09}{12} = 0.0075 \quad (0.75\% \text{ per month})$$

To find the payments,



$$A = \$100,000 (A/P, 0.75\%, 360)$$

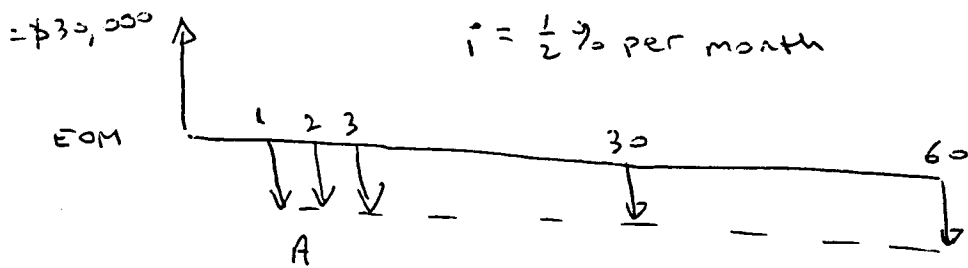
$$= \$100,000 \left[ \frac{0.0075 (1.0075)^{360}}{(1.0075)^{360} - 1} \right]$$

$$= \$100,000 (0.008046226)$$

$$= \$804.62$$

## Calculating The Unpaid Balance on a Loan

ex. A \$30,000 automobile is financed at 6% per year over 5 years. How much is still owed after the 30<sup>th</sup> payment?

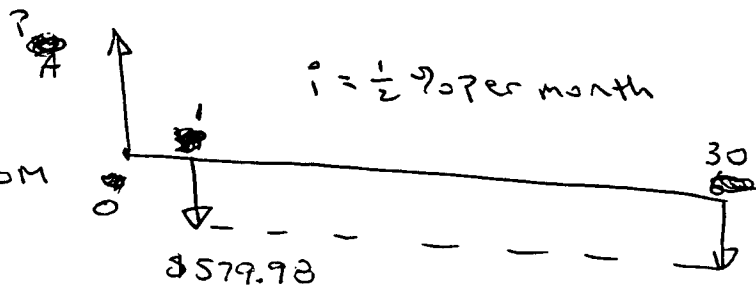


$$A = \$30,000 (A/P \ 0.5\%, 60)$$

$$= \$30,000 \left[ \frac{0.005(1.005)^{60}}{(1.005)^{60} - 1} \right]$$

$$= \$579.98$$

Immediately following the 30<sup>th</sup> payment, the unpaid balance is the PW of the remaining 30 payments.



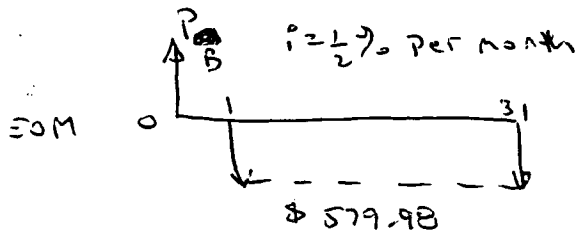
$$P = \$579.98 (P/A \ 0.5\%, 30)$$

$$= \$579.98 \left[ \frac{(1.005)^{30} - 1}{0.005(1.005)^{30}} \right]$$

$$= ~~\$16,375~~ \$16,120.00$$

To find what portion of the 30<sup>th</sup> payment is principal reduction and what portion is interest, it is necessary to find the unpaid balance after the 29<sup>th</sup> payment.

Immediately following the 29th payment,



$$P_B = \$579.98 \text{ (PIA } 0.5\%, 31)$$

$$= \$16,616.89$$

#	Payment	Portion to Principal	Portion to Interest	Unpaid Balance
1	\$579.98	\$429.98	\$150.00	\$29,570.02
29				\$16,616.89
30	\$579.98	\$496.89	\$83.09	\$16,120.00
31	\$579.98	\$499.38	\$80.60	\$15,620.62
60	\$579.98	\$577.09	\$2.89	\$0

Payment 30

$$\text{Principal Reduction} = P_B - P_A$$

$$P_{30} = \$16,616.89 - \$16,120.00$$

$$= \$496.89$$

$$\text{Interest Portion} = A - P_{30}$$

$$I_{30} = \$579.98 - \$496.89$$

$$= \$83.09$$

Checks

$$I_{30} = 0.005(\$16,616.89)$$

$$= \$83.08$$

$$I_{31} = 0.005(\$16,120.)$$

$$= \$80.60$$

$$P_{31} = A - I_{31}$$

$$= \$579.98 - \$80.60$$

$$= \$499.38$$

$$UB_{31} = \$16,120 - \$499.38$$

$$= \$15,620.62$$

$$I_1 = 0.005(\$30,000)$$

$$P_1 = A - I_1$$

$$= \$579.98 - \$150$$

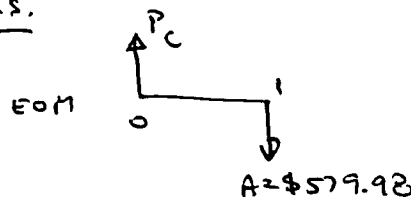
$$= \$429.98$$

$$B_1 = \$30,000 - \$429.98$$

$$= \$29,570.02$$

Question: When portion of last payment (#60) is interest and principal?

Ans.



$$I_{60} = 0.005(\$577.09)$$

$$= \$2.89$$

$$P_{60} = A - I_{60} = \$577.09$$

$$P_C = \$579.98 \text{ (PIA } 0.5\%, 1)$$

$$= \$579.98 \left[ \frac{(1.005)^1 - 1}{0.005(1.005)^1} \right]$$

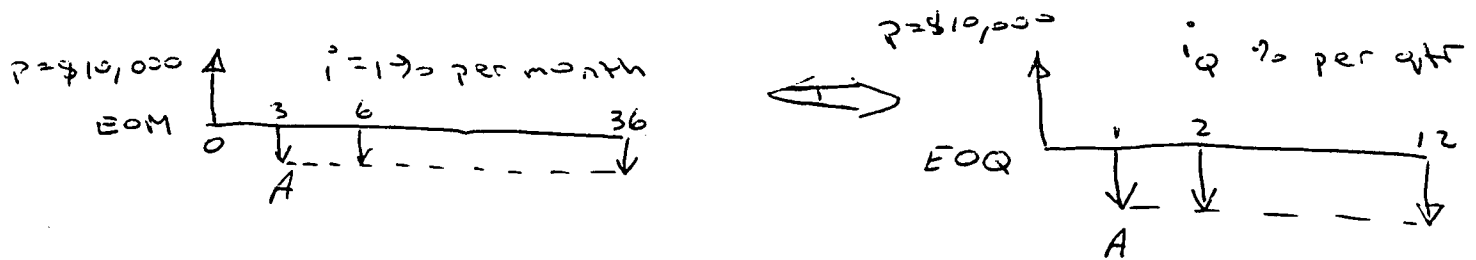
$$= \frac{\$579.98}{1.005} = \$577.09$$

## 4.2 When Interest Periods Coincide with Payment Periods

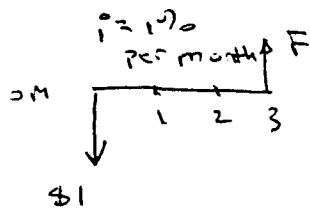
See previous example

## 4.3 When Interest ~~Periods~~ Are Smaller than Payment Periods

ex. Suppose a loan of \$10,000 is to be repaid quarterly for 3 years. Interest is charged at a rate of 1% per month.



To find  $i_q$ , the interest rate per qtr that is equivalent to  $i_m = 1\%$  per month, consider \$1 invested as follows:



$$F = \$1(1 + 0.01)^3$$

$$= \$1.030301$$

$$\therefore i_q = \frac{\$1.030301 - \$1.00}{\$1.00}$$

$$= 0.030301 \text{ (3.0301\% per qtr)}$$

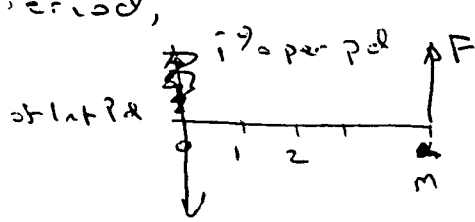
Returning to the problem,

$$A = \$10,000 \left( \frac{A}{P} \mid 3.0301\%, 12 \right)$$

$$= \$10,000 \left[ \frac{0.030301 (1.030301)^{12}}{(1.030301)^{12} - 1} \right]$$

$$= \$1006.43$$

In general, if there are  $m$  interest ~~payments~~ in one payment period,



$$F = P(1+i)^m$$

$$i_{\text{actual}} = \frac{P(1+i)^m - P}{P} = (1+i)^m - 1$$

$$= (1 + \sum_{k=1}^{m-1} i)^m - 1$$

ex. Find the equivalent or actual interest rate per year charged by a credit card company that advertises a nominal interest rate of 13% per yr.

$$i = \frac{r}{n} = \frac{0.13}{12} = 0.0108 \text{ per month}$$

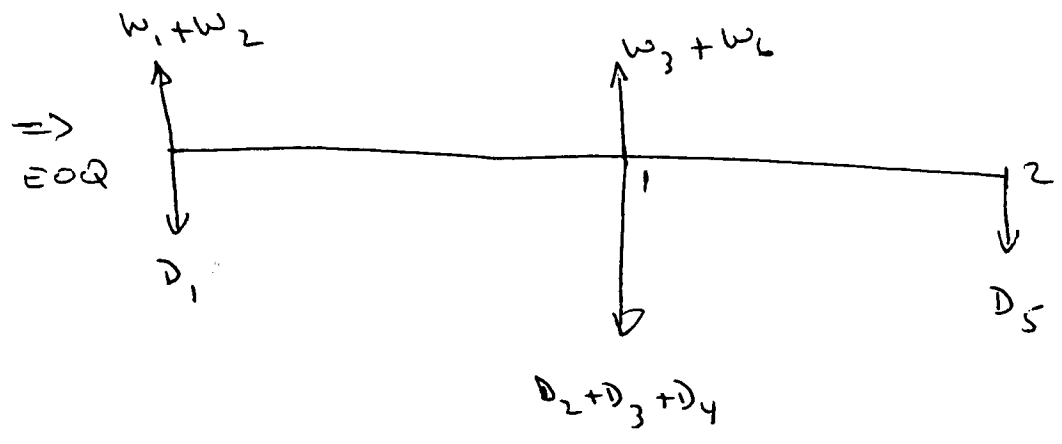
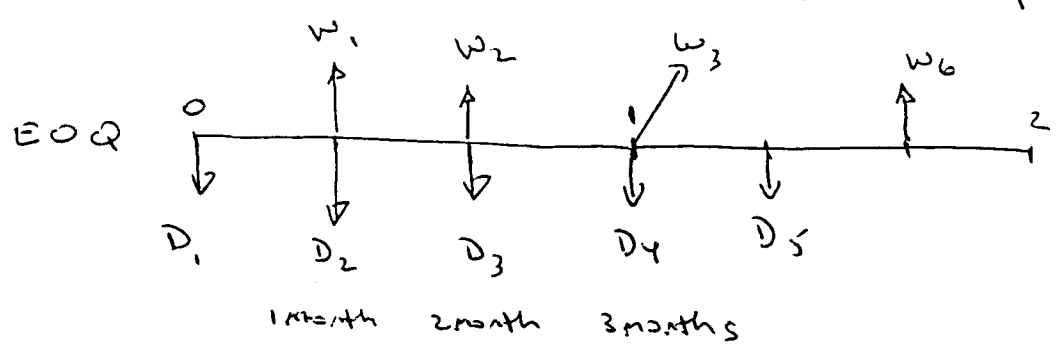
$$i_{\text{actual}} = (1+i)^n - 1$$

$$= (1.0108)^{12} - 1$$

$$= 0.1356182 \quad (13.56\% \text{ per yr})$$

4.4 When Interest Periods Are Larger than Payment Periods

ex. Deposits in a bank acct that pays interest quarterly do not earn interest ~~unless~~ <sup>if</sup> they were deposited after the period started. Withdrawals during an interest period are considered as having occurred at the beginning of the period.



ex. Find the equivalent or actual interest rate per year charged by a credit card company that advertises a nominal interest rate of 18% per yr.

$$i = \frac{r}{m} = \frac{0.18}{12} = 0.015 \text{ per month}$$

$$\begin{aligned} i_{\text{actual}} &= (1+i)^n - 1 \\ &= (1.015)^{12} - 1 \\ &= 0.1956182 \quad (19.56\% \text{ per yr}) \end{aligned}$$

4.4 When Interest Periods Are Larger than Payment Periods

ex. Deposits in a bank acct that pays interest quarterly do not earn interest ~~unless~~ <sup>if</sup> they were deposited after the period started. Withdrawals during an interest period are considered as having occurred at the beginning of the period.

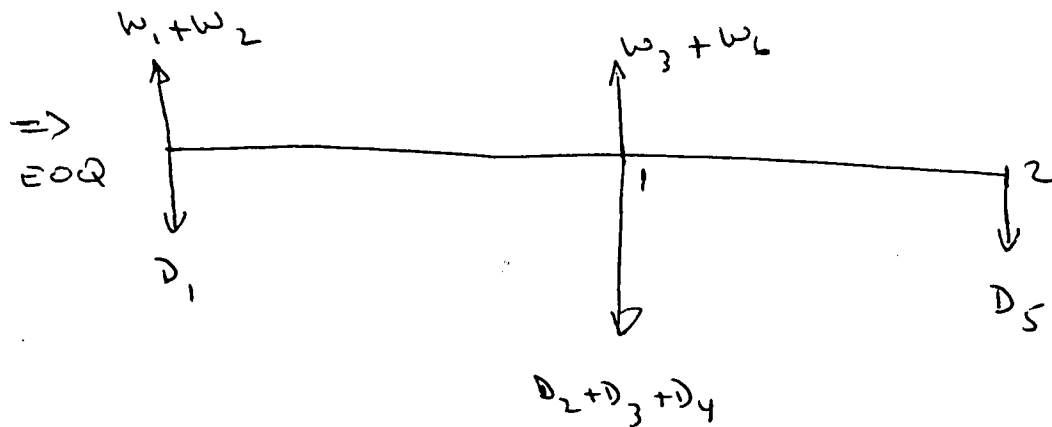
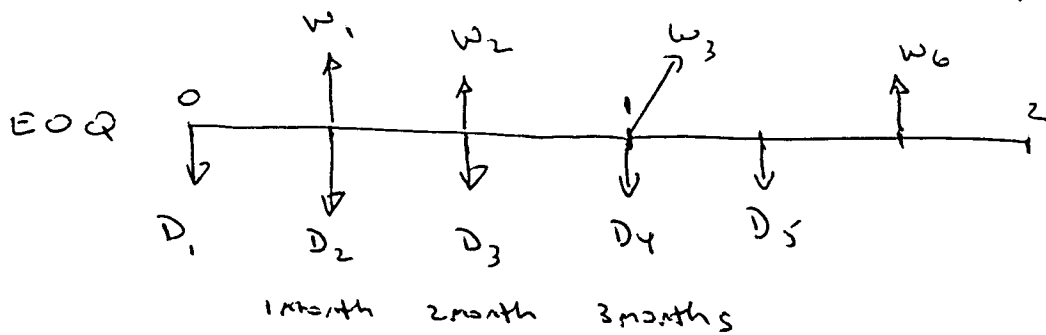


Table C.11 Interest Rate Factors (5.0%)

N	Single Payment		Equal Payment Series				Gradient Series		N
	Compound Amount Factor (F/P,i,N)	Present Worth Factor (P/F,i,N)	Compound Amount Factor (F/A,i,N)	Sinking Fund Factor (A/F,i,N)	Present Worth Factor (P/A,i,N)	Capital Recovery Factor (A/P,i,N)	Gradient Uniform Series (A/G,i,N)	Gradient Present Worth (P/G,i,N)	
1	1.0500	0.9524	1.0000	1.0000	0.9524	1.0500	0.0000	0.0000	1
2	1.1025	0.9070	2.0500	0.4878	1.8594	0.5378	0.4878	0.9070	2
3	1.1576	0.8638	3.1525	0.3172	2.7232	0.3672	0.9675	2.6347	3
4	1.2155	0.8227	4.3101	0.2320	3.5460	0.2820	1.4391	5.1028	4
5	1.2763	0.7835	5.5256	0.1810	4.3295	0.2310	1.9025	8.2269	5
6	1.3401	0.7462	6.8019	0.1470	5.0757	0.1970	2.3579	11.9680	6
7	1.4071	0.7107	8.1420	0.1228	5.7864	0.1728	2.8052	16.2321	7
8	1.4775	0.6768	9.5491	0.1047	6.4632	0.1547	3.2445	20.9700	8
9	1.5513	0.6446	11.0266	0.0907	7.1078	0.1407	3.6758	26.1268	9
10	1.6289	0.6139	12.5779	0.0795	7.7217	0.1295	4.0991	31.6520	10
11	1.7103	0.5847	14.2068	0.0704	8.3064	0.1204	4.5144	37.4988	11
12	1.7959	0.5568	15.9171	0.0628	8.8633	0.1128	4.9219	43.6241	12
13	1.8856	0.5303	17.7130	0.0565	9.3936	0.1065	5.3215	49.9879	13
14	1.9799	0.5051	19.5986	0.0510	9.8986	0.1010	5.7133	56.5538	14
15	2.0789	0.4810	21.5786	0.0463	10.3797	0.0963	6.0973	63.2880	15
16	2.1829	0.4581	23.6575	0.0423	10.8378	0.0923	6.4736	70.1597	16
17	2.2920	0.4363	25.8404	0.0387	11.2741	0.0887	6.8423	77.1405	17
18	2.4066	0.4155	28.1324	0.0355	11.6896	0.0855	7.2034	84.2043	18
19	2.5270	0.3957	30.5390	0.0327	12.0853	0.0827	7.5569	91.3275	19
20	2.6533	0.3769	33.0660	0.0302	12.4622	0.0802	7.9030	98.4884	20
21	2.7860	0.3589	35.7193	0.0280	12.8212	0.0780	8.2416	105.6673	21
22	2.9253	0.3418	38.5052	0.0260	13.1630	0.0760	8.5730	112.8461	22
23	3.0715	0.3256	41.4305	0.0241	13.4886	0.0741	8.8971	120.0087	23
24	3.2251	0.3101	44.5020	0.0225	13.7986	0.0725	9.2140	127.1402	24
25	3.3864	0.2953	47.7271	0.0210	14.0939	0.0710	9.5238	134.2275	25
26	3.5557	0.2812	51.1135	0.0196	14.3752	0.0696	9.8266	141.2585	26
27	3.7335	0.2678	54.6691	0.0183	14.6430	0.0683	10.1224	148.2226	27
28	3.9201	0.2551	58.4026	0.0171	14.8981	0.0671	10.4114	155.1101	28
29	4.1161	0.2429	62.3227	0.0160	15.1411	0.0660	10.6936	161.9126	29
30	4.3219	0.2314	66.4388	0.0151	15.3725	0.0651	10.9691	168.6226	30
31	4.5380	0.2204	70.7608	0.0141	15.5928	0.0641	11.2381	175.2333	31
32	4.7649	0.2099	75.2988	0.0133	15.8027	0.0633	11.5005	181.7392	32
33	5.0032	0.1999	80.0638	0.0125	16.0025	0.0625	11.7566	188.1351	33
34	5.2533	0.1904	85.0670	0.0118	16.1929	0.0618	12.0063	194.4168	34
35	5.5160	0.1813	90.3203	0.0111	16.3742	0.0611	12.2498	200.5807	35
40	7.0400	0.1420	120.7998	0.0083	17.1591	0.0583	13.3775	229.5452	40
45	8.9850	0.1113	159.7002	0.0063	17.7741	0.0563	14.3644	255.3145	45
50	11.4674	0.0872	209.3480	0.0048	18.2559	0.0548	15.2233	277.9148	50
55	14.6356	0.0683	272.7126	0.0037	18.6335	0.0537	15.9664	297.5104	55
60	18.6792	0.0535	353.5837	0.0028	18.9293	0.0528	16.6062	314.3432	60
65	23.8399	0.0419	456.7980	0.0022	19.1611	0.0522	17.1541	328.6910	65
70	30.4264	0.0329	588.5285	0.0017	19.3427	0.0517	17.6212	340.8409	70
75	38.8327	0.0258	756.6537	0.0013	19.4850	0.0513	18.0176	351.0721	75
80	49.5614	0.0202	971.2288	0.0010	19.5965	0.0510	18.3526	359.6460	80
85	63.2544	0.0158	1245.0871	0.0008	19.6838	0.0508	18.6346	366.8007	85
90	80.7304	0.0124	1594.6073	0.0006	19.7523	0.0506	18.8712	372.7488	90
95	103.0347	0.0097	2040.6935	0.0005	19.8059	0.0505	19.0689	377.6774	95
100	131.5013	0.0076	2610.0252	0.0004	19.8479	0.0504	19.2337	381.7492	100

Table C.24 Interest Rate Factors (20.0%)

N	Single Payment		Equal Payment Series				Gradient Series		N
	Compound Amount Factor (F/P,i,N)	Present Worth Factor (P/F,i,N)	Compound Amount Factor (F/A,i,N)	Sinking Fund Factor (A/F,i,N)	Present Worth Factor (P/A,i,N)	Capital Recovery Factor (A/P,i,N)	Gradient Uniform Series (A/G,i,N)	Gradient Present Worth (P/G,i,N)	
1	1.2000	0.8333	1.0000	1.0000	0.8333	1.2000	0.0000	0.0000	1
2	1.4400	0.6944	2.2000	0.4545	1.5278	0.6545	0.4545	0.6944	2
3	1.7280	0.5787	3.6400	0.2747	2.1065	0.4747	0.8791	1.8519	3
4	2.0736	0.4823	5.3680	0.1863	2.5887	0.3863	1.2742	3.2986	4
5	2.4883	0.4019	7.4416	0.1344	2.9906	0.3344	1.6405	4.9061	5
6	2.9860	0.3349	9.9299	0.1007	3.3255	0.3007	1.9788	6.5806	6
7	3.5832	0.2791	12.9159	0.0774	3.6046	0.2774	2.2902	8.2551	7
8	4.2998	0.2326	16.4991	0.0606	3.8372	0.2606	2.5756	9.8831	8
9	5.1598	0.1938	20.7989	0.0481	4.0310	0.2481	2.8364	11.4335	9
10	6.1917	0.1615	25.9587	0.0385	4.1925	0.2385	3.0739	12.8871	10
11	7.4301	0.1346	32.1504	0.0311	4.3271	0.2311	3.2893	14.2330	11
12	8.9161	0.1122	39.5805	0.0253	4.4392	0.2253	3.4841	15.4667	12
13	10.6993	0.0935	48.4966	0.0206	4.5327	0.2206	3.6597	16.5883	13
14	12.8392	0.0779	59.1959	0.0169	4.6106	0.2169	3.8175	17.6008	14
15	15.4070	0.0649	72.0351	0.0139	4.6755	0.2139	3.9588	18.5095	15
16	18.4884	0.0541	87.4421	0.0114	4.7296	0.2114	4.0851	19.3208	16
17	22.1861	0.0451	105.9306	0.0094	4.7746	0.2094	4.1976	20.0419	17
18	26.6233	0.0376	128.1167	0.0078	4.8122	0.2078	4.2975	20.6805	18
19	31.9480	0.0313	154.7400	0.0065	4.8435	0.2065	4.3861	21.2439	19
20	38.3376	0.0261	186.6880	0.0054	4.8696	0.2054	4.4643	21.7395	20
21	46.0051	0.0217	225.0256	0.0044	4.8913	0.2044	4.5334	22.1742	21
22	55.2061	0.0181	271.0307	0.0037	4.9094	0.2037	4.5941	22.5546	22
23	66.2474	0.0151	326.2369	0.0031	4.9245	0.2031	4.6475	22.8867	23
24	79.4968	0.0126	392.4842	0.0025	4.9371	0.2025	4.6943	23.1760	24
25	95.3962	0.0105	471.9811	0.0021	4.9476	0.2021	4.7352	23.4276	25
26	114.4755	0.0087	567.3773	0.0018	4.9563	0.2018	4.7709	23.6460	26
27	137.3706	0.0073	681.8528	0.0015	4.9636	0.2015	4.8020	23.8353	27
28	164.8447	0.0061	819.2233	0.0012	4.9697	0.2012	4.8291	23.9991	28
29	197.8136	0.0051	984.0680	0.0010	4.9747	0.2010	4.8527	24.1406	29
30	237.3763	0.0042	1181.8816	0.0008	4.9789	0.2008	4.8731	24.2628	30
31	284.8516	0.0035	1419.2579	0.0007	4.9824	0.2007	4.8908	24.3681	31
32	341.8219	0.0029	1704.1095	0.0006	4.9854	0.2006	4.9061	24.4588	32
33	410.1863	0.0024	2045.9314	0.0005	4.9878	0.2005	4.9194	24.5368	33
34	492.2235	0.0020	2456.1176	0.0004	4.9898	0.2004	4.9308	24.6038	34
35	590.6682	0.0017	2948.3411	0.0003	4.9915	0.2003	4.9406	24.6614	35
40	1469.7716	0.0007	7343.8578	0.0001	4.9966	0.2001	4.9728	24.8469	40
45	3657.2620	0.0003	18281.3099	0.0001	4.9986	0.2001	4.9877	24.9316	45

Table C.16 Interest Rate Factors (10.0%)

N	Single Payment		Equal Payment Series				Gradient Series		N
	Compound Factor (F/P,i,N)	Present Worth Factor (P/F,i,N)	Compound Amount Factor (F/A,i,N)	Sinking Fund Factor (A/F,i,N)	Present Worth Factor (P/A,i,N)	Capital Recovery Factor (A/P,i,N)	Gradient Uniform Series (A/G,i,N)	Gradient Present Worth (P/G,i,N)	
1	1.1000	0.9091	1.0000	1.0000	0.9091	1.1000	0.0000	0.0000	1
2	1.2100	0.8264	2.1000	0.4762	1.7355	0.5762	0.4762	0.8264	2
3	1.3310	0.7513	3.3100	0.3021	2.4869	0.4021	0.9366	2.3291	3
4	1.4641	0.6830	4.6410	0.2155	3.1699	-0.3155	1.3812	4.3781	4
5	1.6105	0.6209	6.1051	0.1638	3.7908	0.2638	1.8101	6.8618	5
6	1.7716	0.5645	7.7156	0.1296	4.3553	0.2296	2.2236	9.6842	6
7	1.9487	0.5132	9.4872	0.1054	4.8684	0.2054	2.6216	12.7631	7
8	2.1436	0.4665	11.4359	0.0874	5.3349	0.1874	3.0045	16.0287	8
9	2.3579	0.4241	13.5795	0.0736	5.7590	0.1736	3.3724	19.4215	9
10	2.5937	0.3855	15.9374	0.0627	6.1446	0.1627	3.7255	22.8913	10
11	2.8531	0.3505	18.5312	0.0540	6.4951	0.1540	4.0641	26.3963	11
12	3.1384	0.3186	21.3843	0.0468	6.8137	0.1468	4.3884	29.9012	12
13	3.4523	0.2897	24.5227	0.0408	7.1034	0.1408	4.6988	33.3772	13
14	3.7975	0.2633	27.9750	0.0357	7.3667	0.1357	4.9955	36.8005	14
15	4.1772	0.2394	31.7725	0.0315	7.6061	0.1315	5.2789	40.1520	15
16	4.5950	0.2176	35.9497	0.0278	7.8237	0.1278	5.5493	43.4164	16
17	5.0545	0.1978	40.5447	0.0247	8.0216	0.1247	5.8071	46.5819	17
18	5.5599	0.1799	45.5992	0.0219	8.2014	0.1219	6.0526	49.6395	18
19	6.1159	0.1635	51.1591	0.0195	8.3649	0.1195	6.2861	52.5827	19
20	6.7275	0.1486	57.2750	0.0175	8.5136	0.1175	6.5081	55.4069	20
21	7.4002	0.1351	64.0025	0.0156	8.6487	0.1156	6.7189	58.1095	21
22	8.1403	0.1228	71.4027	0.0140	8.7715	0.1140	6.9189	60.6893	22
23	8.9543	0.1117	79.5430	0.0126	8.8832	0.1126	7.1085	63.1462	23
24	9.8497	0.1015	88.4973	0.0113	8.9847	0.1113	7.2881	65.4813	24
25	10.8347	0.0923	98.3471	0.0102	9.0770	0.1102	7.4580	67.6964	25
26	11.9182	0.0839	109.1818	0.0092	9.1609	0.1092	7.6186	69.7940	26
27	13.1100	0.0763	121.0999	0.0083	9.2372	0.1083	7.7704	71.7773	27
28	14.4210	0.0693	134.2099	0.0075	9.3066	0.1075	7.9137	73.6495	28
29	15.8631	0.0630	148.6309	0.0067	9.3696	0.1067	8.0489	75.4146	29
30	17.4494	0.0573	164.4940	0.0061	9.4269	0.1061	8.1762	77.0766	30
31	19.1943	0.0521	181.9434	0.0055	9.4790	0.1055	8.2962	78.6395	31
32	21.1138	0.0474	201.1378	0.0050	9.5264	0.1050	8.4091	80.1078	32
33	23.2252	0.0431	222.2515	0.0045	9.5694	0.1045	8.5152	81.4856	33
34	25.5477	0.0391	245.4767	0.0041	9.6086	0.1041	8.6149	82.7773	34
35	28.1024	0.0356	271.0244	0.0037	9.6442	0.1037	8.7086	83.9872	35
40	45.2593	0.0221	442.5926	0.0023	9.7791	0.1023	9.0962	88.9525	40
45	72.8905	0.0137	718.9048	0.0014	9.8628	0.1014	9.3740	92.4544	45
50	117.3909	0.0085	1163.9085	0.0009	9.9148	0.1009	9.5704	94.8889	50
55	189.0591	0.0053	1880.5914	0.0005	9.9471	0.1005	9.7075	96.5619	55
60	304.4816	0.0033	3034.8164	0.0003	9.9672	0.1003	9.8023	97.7010	60
65	490.3707	0.0020	4893.7073	0.0002	9.9796	0.1002	9.8672	98.4705	65
70	789.7470	0.0013	7887.4696	0.0001	9.9873	0.1001	9.9113	98.9870	70
75	1271.8954	0.0008	12708.9537	0.0001	9.9921	0.1001	9.9410	99.3317	75
80	2048.4002	0.0005	20474.0021	0.0000	9.9951	0.1000	9.9609	99.5606	80
85	3298.9690	0.0003	32979.6903	0.0000	9.9970	0.1000	9.9742	99.7120	85
90	5313.0226	0.0002	53120.2261	0.0000	9.9981	0.1000	9.9831	99.8118	90
95	8556.6760	0.0001	85556.7605	0.0000	9.9988	0.1000	9.9889	99.8773	95
100	13780.6123	0.0001	137796.1234	0.0000	9.9993	0.1000	9.9927	99.9202	100